6.7

a) $\int_1^e \ln x \, dx = \int_1^e 1 \cdot \ln x \, dx =$

$\int u' \cdot v = uv - \int uv'$

$u' = 1 \qquad v = \ln x$

$u = x \qquad v' = \frac{1}{x}$

$= \left[x \cdot \ln x \right]_1^e - \int_1^e \left(x \cdot \frac{1}{x} \right) dx = e \cdot \underbrace{\ln e}_{1} - 1 \cdot \underbrace{\ln 1}_{0} - \int_1^e 1 \cdot dx =$

$= e - \left[x \right]_1^e = e - (e - 1) = \underline{\underline{1}}$

b) $\int_1^2 \underbrace{(3x+2)}_{u'} \cdot \underbrace{\ln x}_{v} \, dx = \left[\left(\frac{3x^2}{2} + 2x \right) \cdot \overset{\ln x}{\cancel{\frac{1}{x}}} \right]_1^2 - \int_1^2 \left[\left(\frac{3x^2}{2} + 2x \right) \cdot \frac{1}{x} \, dx =$

$u' = 3x + 2 \qquad v = \ln x$

$u = \frac{3x^2}{2} + 2x \qquad v' = \frac{1}{x}$

$= \left(\left(\frac{3 \cdot 2^2}{2} + 2 \cdot 2 \right) \cdot \ln 2 - \left(\frac{3 \cdot 1^2}{2} + 2 \cdot 1 \right) \cdot \underbrace{\ln 1}_{0} \right) - \int_1^2 \left(\frac{3x}{2} + 2 \right) dx =$

$= 10 \ln 2 - 0 - \left[\frac{3x^2}{4} + 2x \right]_1^2 = 10 \ln 2 - \left[\left(\frac{3 \cdot 2^2}{4} + 4 \right) - \left(\frac{3 \cdot 1^2}{4} + 2 \right) \right] =$

$= 10 \ln 2 - \left(7 - \frac{3}{4} - 2 \right) = 10 \ln 2 - 4\frac{1}{4} =$

$= \underline{\underline{10 \ln 2 - \frac{17}{4}}}$

6.7

c) $\int_1^e x^3 \cdot \ln x \, dx =$

$\int u' v = uv - \int u v'$

$u' = x^3 \quad v = \ln x$

$u = \frac{x^4}{4} \quad v' = \frac{1}{x}$

$= \left[\frac{x^4}{4} \cdot \ln x\right]_1^e - \int_1^e \left(\frac{x^4}{4} \cdot \frac{1}{x}\right) dx = \left[\frac{e^4}{4} \cdot \underbrace{\ln e}_{1} - \frac{1^4}{4} \cdot \underbrace{\ln 1}_{0}\right] - \int_1^e \frac{x^3}{4} dx$

$= \frac{e^4}{4} - 0 - \left[\frac{1}{4} \cdot \frac{x^4}{4}\right]_1^e = \frac{e^4}{4} - \left(\frac{e^4}{16} - \frac{1}{16}\right) =$

$= \frac{4e^4 - e^4}{16} + \frac{1}{16} = \frac{3e^4}{16} + \frac{1}{16} = \frac{3e^4 + 1}{16}$

d) $\int_0^{\frac{\pi}{2}} x \cos x \, dx =$

$\int u \cdot v' = u \cdot v - \int u' v$

$u = x \quad v' = \cos x$

$u' = 1 \quad v = \sin x$

$= \left[x \cdot \sin x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x \, dx = \left(\frac{\pi}{2} \cdot \sin\frac{\pi}{2} - 0 \cdot \sin 0\right) - \left[-\cos x\right]_0^{\frac{\pi}{2}} =$

$= \frac{\pi}{2} - \left(-\underbrace{\cos\frac{\pi}{2}}_{0} + \underbrace{\cos 0}_{1}\right) = \frac{\pi}{2} - (0 + 1) = \frac{\pi}{2} - 1$